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Analysis and comparison of methods of risk-free rate estimation

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Abstract: Capital Asset Pricing Model is nowadays the most applied model for the valuation of capital assets. This paper is devoted to the issue of determining the value of risk-free rate in CAPM. The basic issue is the accuracy of risk-free rate determination from the past values of return on government bonds compared with the current rate of return. The aim of the paper was to analyse various methods of estimating risk-free rate, to compare them and to formulate recommendations on risk-free rate determination. When solving the research problem, risk-free rate was estimated applying arithmetic and geometric average of historical returns on bonds, using regression analysis based on historical data and applying direct estimation into the future. For the comparison, the values of return on Slovak government bonds and US T. Bonds, as well as values of Slovak zero bonds, were used. We can conclude that return on government bonds based on direct estimate is lower than return on bonds calculated as the average of historical data. Based on this finding it can be recommended to apply current return on government bonds in the valuation of capital assets. This rate of return will not increase the value of assets.
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Keywords: CAPM, government bonds, risk-free rate

JEL Classification: G32

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1. Introduction

Capital Asset Pricing Model belongs nowadays to one of the most commonly used models for the valuation of capital assets. Jindřichovská (2013) states that the history of CAPM goes back to the 50s and this model is based on Harry Markowitz portfolio model. Markovitz work became the basis for the papers of Jack Treynor (1961, 1962), William Sharpe (1964), John Lintner (1965) and Jan Mossin (1966), in which they dealt with the portfolio theory and risk diversification. CAPM has become a basic pillar in the area of financial economics, currently it is one of the most frequently applied asset pricing models and it is still considered to be the important part of modern finance. Sivák, Gertler and Kováč (2010) states that in the CAPM equation, the expected return on individual portfolio should be equal to the sum of risk-free rate and Risk Premium, which depends on the nature of stock „*i*". Character of the stock in this model is determined by „*β*" coefficient.

2. Literature review

The formula for the World CAPM model is according to its founders (Fama, French 2004; Saita 2007; Kislingerová 2010; Klieštik, Valašková 2013) as follows: $E(r_i) = r_f + \beta * [E(r_m) - (r_f)]$, *where E(ri) – Expected Return on the Capital Asset, $E(r_m)$ - Expected Return on the Market, r_f - Risk-free rate, $[E(r_m) - (r_f)]$ = ERP (Equity Risk Premium) - indicates how much is the expected return on stocks in the market higher than the return on risk-free investment, β – beta coefficient - measures the asset`s individual exposure to market risk, is the sensitivity of the expected excess asset returns to the expected excess market returns.*

According to Pavelková and Knápková (2005) to use CAPM for the assets and equity valuation, we have to be able to determine these inputs: Risk-free rate - r_f, Equity Risk Premium – *ERP*, *β* coefficient - *β* for specific industry, resp. sector.

This paper is focused on the estimation of risk-free rate - r_f,. In this regard we set these research questions: *Which value of risk-free rate is valid in terms of valuation practice in Slovakia? Is it more appropriate to use average value of return on government bonds from the past or to focus on its direct estimate to the future?*

We can generally say that absolutely risk-free assets do not exist. In the US, treasury bills – US T. Bills are considered the least risky. Though, valuation based on their rate of return is used when valuating individual shares. For the valuation of Cost of Equity of the enterprise, it is recommended to use the rate of return on 10-year Slovak T. Bonds (NBS 2016). Mařík et al. (2011) also considers it necessary to determine return on government bonds with longer maturity – 10 years or more. If there is a number of government bonds at the market with a similar maturity, it is appropriate to use average return to maturity.

In these days academics (Damodaran 2001; Drukarczyk 2003; Mařík 2011) base the value of r_f on the current return on of long-term government bonds. But their approach is not the only one that can be used in the valuation practice. When choosing specific methods for determining risk-free rate, it should be taken into account in particular the period to which the return is related.

From the principles of revenue valuation results that it should be a forecast of future risk-free rate. In this case r_f prediction is based on the past performance of government bonds or on direct view into the future.

<u>*Prediction of r_f based on the past return on government bonds:*</u> Applying averages of past values of stock return is widely used in current practice. The main reason for the application of this method of forecasting the risk-free rate is the possibility of excluding the impact of fluctuations of return on government bonds on company valuation, which can be considered as a major disadvantage of using current return. In the case that data on stock return are available for a longer period of time, we can consider the average return as an objective variable for determining risk-free rate. However, this way of calculation also has disadvantages. Despite the fact that this approach is considered to be objective, it is possible to point out the significant differences in the valuation of assets, namely when using arithmetic or geometric average (Table 4 and Table 5). We can say that using the average of past returns may not be a good estimate of the future returns, especially for market valuation of the company. In these cases, the estimate based on direct insight into the future is more appropriate.

To propose r_f based on the direct insight into the future, we have three options (Mařík 2011):

a) Forecast of interest rates based on common forecasting system, it should be pointed out that the forecasts of individual prognostic companies vary significantly and can serve as a support tool for the valuation and forecast of r_f.
b) Current return to maturity of government bonds, while it is recommended to use bonds with the longest remaining maturity. In this case the problem can be defined as follows: What will be the value of r_f after the repayment of bond?
c) Spot and other interest rates in capital market. These interest rates create conditions for the application of differentiated risk-free premiums each year.

In the opinion of Mařík et al. (2011) forecast of interest rates using forecasting tools can be used as a support mechanism for the estimation of risk-free rate r_f. The most common method to determine r_f is the method of current return on government bonds. This method is also the most common method of valuation in today`s practice, used instead of application of interest rates. The spot interest rate represents interest rate on the deposit for the period of time, without the payment of interest. This rate of return corresponds to zero bonds maturing in that period. According to Mařík (2011) to estimate risk-free rate, interest rates of government zero-coupon bonds should be used. It is important to say that spot interest rates are not constant. The usual shape of the yield curve is upward sloping and since spot interest rates have a certain time structure, they can be used to construct yield curve.

In terms of the research problem and research questions we set the aim of the paper: To estimate value of „r_f“ applying selected methods (forecast of interest rates based on forecasting tools, estimation using current rate of return on government bonds, prognosis with the application of geometric or arithmetic average of historical returns on bonds, estimate using spot interest rate) and to compare results obtained.

There are several methods for determining spot rate available in today's valuation practice. From these methods, we applied method of Swensson (Málek, Radová, Šťerba 2007; Mařík 2011). This method is recommended also by experts from Germany, Swensson derived following formula (Málek, Radová, Šťerba 2007: Mařík 2011):

$$z(T,\beta,\tau) = \beta_0 + \beta_1 * \left(\frac{1 - e^{\frac{-T}{\tau_1}}}{\frac{T}{\tau_1}} \right) + \beta_2 * \left(\frac{1 - e^{\frac{-T}{\tau_1}}}{\frac{T}{\tau_1}} - e^{\frac{-T}{\tau_1}} \right) + \beta_3 * \left(\frac{1 - e^{\frac{-T}{\tau_2}}}{\frac{T}{\tau_2}} - e^{\frac{-T}{\tau_2}} \right)$$

where

$z(T,\beta,\tau)$ - *interest rate for hypothetical zero bonds maturing in period T,*
$\beta_{0,1,2,3}$ - *function parameters,*
$\tau_{1,2}$ - *function parameters.*

Calculation of individual parameters is difficult; however Ministry of Finance of the Slovak Republic performs their daily calculations which are available at www.finance.gov.sk.

In accordance with research problem we set scientific hypothesis:

H: We assume that the value of risk-free rate „r_f“ in the case of its estimate using past data will be higher than in the case of applying current rate of return.

3. Results and discussion

At the beginning of this chapter we discuss the possibilities of obtaining inputs for the calculation of risk-free rate - „r_f“. US T. Bond Rate can be found on the website of prof. Damodaran (*http://pages.stern.nyu.edu/~adamodar/* - in the structure according to Table 1). The rate of return on Slovak government bonds can be found on the website of National Bank of Slovakia (*www.nbs.sk/* - in the structure according to Table 2).

Table 1. Annual returns on US investments

Year	S&P 500	Return on T.Bill	Return on Bonds	T. Bond Rate
1990	-3.06%	7.55%	6.24%	8.07%
1991	30.23%	5.61%	15.00%	6.70%
1992	7.49%	3.41%	9.36%	6.68%
1993	9.97%	2.98%	14.21%	5.79%
1994	1.33%	3.99%	-8.04%	7.82%
1995	37.20%	5.52%	23.48%	5.57%
1996	22.68%	5.02%	1.43%	6.41%
1997	33.10%	5.05%	9.94%	5.74%
1998	28.34%	4.73%	14.92%	4.65%
1999	20.89%	4.51%	-8.25%	6.44%
2000	-9.03%	5.76%	16.66%	5.11%
2001	-11.85%	3.67%	5.57%	5.05%
2002	-21.97%	1.66%	15.12%	3.81%
2003	28.36%	1.03%	0.38%	4.25%
2004	10.74%	1.23%	4.49%	4.22%
2005	4.83%	3.01%	2.87%	4.39%
2006	15.61%	4.68%	1.96%	4.70%
2007	5.48%	4.64%	10.21%	4.02%
2008	-36.55%	1.59%	20.10%	2.21%
2009	25.94%	0.14%	-11.12%	3.84%
2010	14.82%	0.13%	8.46%	3.29%
2011	2.10%	0.03%	16.04%	1.88%
2012	15.89%	0.05%	2.97%	1.76%
2013	32.15%	0.07%	-9.10%	3.04%
2014	13.52%	0.05%	10.75%	2.17%
2015	1.36%	0.21%	1.28%	2.27%

Source: Damodaran 2016

US T. Bond Rate had in recent years fluctuating tendency, however nowadays it achieves low values. In 2015 this rate achieved 2.27%.

Table 2. Annual return on Slovak government bonds

Year	2003	2004	2005	2006	2007	2008	2009
Return on Slovak government bonds	4.99%	5.02%	3.52%	4.41%	4.49%	4.72%	4.71%
Year	2010	2011	2012	2013	2014	2015	
Return on Slovak government bonds	3.87%	4.45%	4.55%	3.10%	2.07%	0.89%	

Source: NBS 2016

If we compare rate of return on Slovak government bonds and US T. Bond Rate, we can see that in 2011 and 2012 return on Slovak government bonds achieved approximately 4%, while US T. Bond Rate was lower than 2%. In the years 2013 and 2014 the return on Slovak government bonds and US T. Bond Rate were approximately the same.

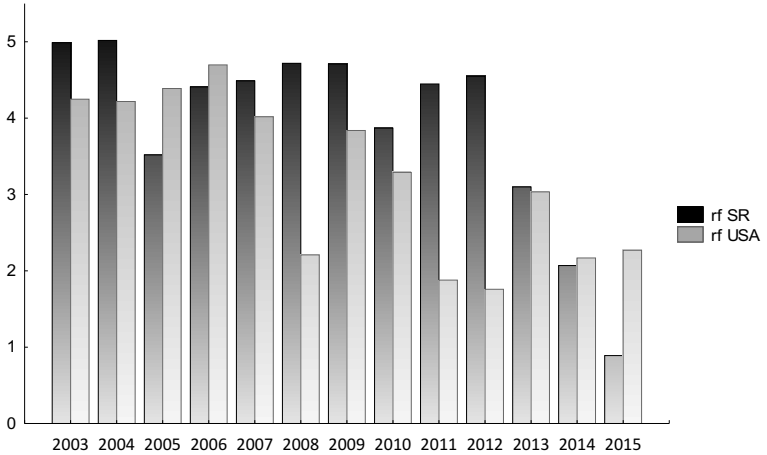

Figure 1. Comparison of return on 10-year government bonds in Slovakia and USA (in %)
Source: Authors; Damodaran 2016

In 2015 there was historic fall of return on Slovak government bonds (see Table 2 and Table 3) - until the end of 2015 the returns decreased below 1%. In contrary, the return on US T. Bonds in 2015 increased slightly (see Table 1). This affects also the equity valuation of companies.

Table 3. Monthly rate of return on Slovak government bonds in 2015

Period	Return on 2-year government bonds	Return on 5-year government bonds	Return on 10-year government bonds
I.15	-0.39%	0.55%	1.22%
II.15	-0.90%	0.57%	1.21%
III.15	-1.09%	0.17%	1.19%
IV.15	-1.32%	0.01%	1.18%
V.15	-1.54%	0.05%	0.74%
VI.15	-1.77%	-0.11%	0.50%
VII.15	-2.05%	-0.17%	0.47%
VIII.15	-2.35%	-0.24%	0.99%
IX.15	-2.69%	-0.31%	0.89%
X.15	-3.05%	-0.38%	0.75%
XI.15	-0.44%	-0.45%	0.71%
XII.15	-0.57%	-0.53%	0.72%

Source: NBS 2016

If we based estimation of „r_f“ on the value of rate of return predicted with the use of historical data, we would get almost negative values of risk-free rate. Therefore it is more appropriate to use current return on government bonds in equity valuation.

Slightly more favourable values of return are obtained from arithmetic and geometric averages but their application would lead to overvaluation of Cost of Equity. If we compare arithmetic average of return on US T. Bonds for the period 2006-2015 with their current return, the difference between them is 2.89%. And if we compare geometric average for the same period with current return on US T. Bonds, the difference is 2.44%. Arithmetic

average of return on Slovak government bonds is 3.73%, while it exceeds the current return by 3.02%. Geometric average of return on Slovak government bonds is 3.38%, it is higher than current return by 2.67%.

In Table 4 is an overview of arithmetic average of return on US T. Bonds.

Table 4. Arithmetic average of return on US T. Bonds

Arithmetic average	3-month T.Bill	10-year T. Bond
1928-2015	3.49%	5.23%
1966-2015	4.97%	7.12%
2006-2015	1.16%	5.16%

Source: Damodaran 2016

Table 5 shows geometric average of return on US T. Bonds. As we can see values of geometric average are lower than values of the arithmetic one.

Table 5. Geometric average of return on US T. Bonds

Geometric average	3-month T.Bill	10-year T. Bond
1928-2015	3.45%	4.96%
1966-2015	4.92%	6.71%
2006-2015	1.14%	4.71%

Source: Damodaran 2016

To predict return on Slovak government bonds and US T. Bonds, we applied regression analysis. When modelling time series in order to forecast future values, we used quadratic equation. Estimation of the parameters of a polynomial equation was carried out in Microsoft Excel on the basis of the values of time series using the method of least squares.

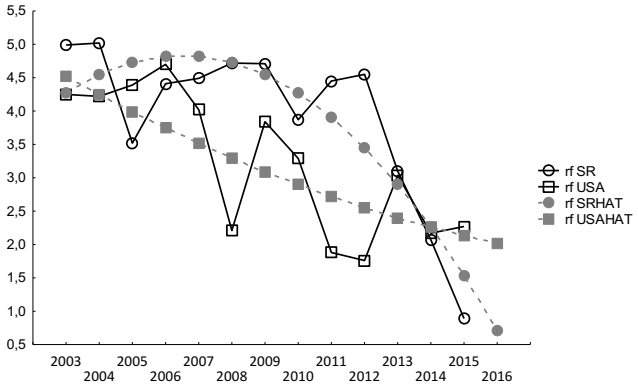

Figure 2. Prediction of return on 10-year goverment bonds in Slovakia and USA – (in %)
Source: Authors

For the purpose of regression analysis we used historical data of the return on Slovak government bonds for the period 2003-2015 (line rf SR on Figure 2) and historical data of the return on US T. Bonds for the period 1999-2015 (line rf USA on Figure 2). We visualized regression curves on the Figure 2 (line rf SRHAT and line rf USAHAT). Applying regression analysis we forecasted the development of return on Slovak government bonds for the year 2016 at the level of 0.71%. We also predicted the development of return on US T. Bonds for the year 2016 at the level of 2.02%.

With the use of Swensson method we calculated spot interest rates for a period of 15 years (see Figure 3). Values of these interest rates show upward trend.

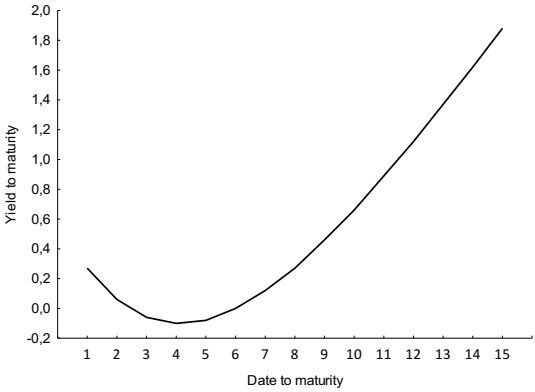

Figure 3. Estimated yield curve
Source: Authors

4. Conclusions

Table 6 summarizes options for estimating risk-free rate - r_f. The lowest value achieved current return on Slovak government bonds in January 2016. This value is the same as the forecast of return on Slovak government bonds, which was calculated applying regression analysis based on historical data of return on Slovak government bonds. Significantly higher is the current return on US T. Bonds (2.27%). This value is higher than forecast of return on US T. Bonds calculated with the use of regression analysis by 0.25%.

Table 6. Comparison of risk-free rate for 2016

Way of calculation	Value
Current return on Slovak government bonds	0.71%
Current return on US government bonds	2.27%
Arithmetic average of return on US government bonds (2006-2015)	5.16%
Geometric average of return on US government bonds (2006-2015)	4.71%
Arithmetic average of return on Slovak government bonds (2006-2015)	3.73%
Geometric average of return on Slovak government bonds (2016-2015)	3.38%
The forecast of return on Slovak government bonds applying regression analysis	0.71%
The forecast of return on US government bonds applying regression analysis	2.02%

Source: Authors; Damodaran 2016; NBS 2016

If we look at values of averages of return on Slovak and US government bonds, these are higher than current returns. Based on these calculations, it is possible to recommend using current return on government bonds in equity valuation applying CAPM. The use of this return will not artificially increase Equity value and it will also eliminate the impact of past returns in the current Equity valuation. However, this method is suitable in the case of flat yield curves. Based on these calculations we can conclude that hypothesis „*We assume that the value of risk-free rate „r_f“ in the case of its estimate using past data will be higher than in the case of applying current rate of return” was confirmed.*

At the end we can summarize some recommendations for the estimation of risk-free rate: This rate should be based primarily on forecasted values. Average of past values can be appropriate starting point for risk-free rate estimation. It is also recommended to base the estimation of risk-free rate on market data, despite the fact that quality

of these data is not appropriate for precise forecast of future development. Equally discussed is the issue of applying differentiated risk-free rate. In practice is currently applied undifferentiated interest rate. The spot interest rates are considered to be an appropriate method for estimation of differentiated interest rates.

References

Copeland, T., T. Koller and J. Murrin. 2000. *Valuation – Measuring and Managing the Value of Companies*. 3rd Ed. New York: Wiley.

Damodaran, A. 2001. "The Dark Side of Valuation." Accessed January 1, 2016. http://pages.stern.nyu.edu/~adamodar/.

Damodaran, A. 2016. "*The data page.*" Accessed January 25, 2016. http://pages.stern.nyu.edu/~adamodar/.

Drukarczyk, J. 2003. *Untrnehmensbewertung.* 4. Auflage. München: Verlag Vahlen.

Fama, E.F. and K. R. French. 2004. "The Capital Assets Pricing Model: Theory and Evidence." *Journal of Economic Perspectives* 18(3), 25-46. Accessed May 15, 2015. http://www1.american.edu/academic.depts/ksb/finance_realestate/mrobe/Library/capm_Fama_French_JEP04.pdf.

Chajdiak, J. 2009. *Štatistika v Exceli 2007.* Bratislava: Statis.

Jindřichovská, I. 2013. *Finanční management.* 1.vyd. Praha: C.H.BECK.

Kislingerová, E. a kol. 2010. *Manažerské finance*. 3. vyd. Praha: C. H. Beck.

Klieštik, T. a K. Valašková. 2013. "Metodológia odhadu koeficienta beta modelu CAPM pomocou Kalmanovho filtra." *Podniková ekonomika a manažment* 1, 16-22. Accessed March 2, 2015. ISSN 1336-5878. http://ke.uniza.sk/assets/casopis/Cislo12013.pdf.

Lintner, J. 1965. "The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets." R*eview of Economics and Statistics* 47(1), 13-37.

Málek, J., J. Radová a F. Štěrba. 2007. "Konstrukce výnosové křivky pomocí vládních dluhopisú v České republice." *Politická ekonomie* 6, 792-808. ISSN 0032-3233.

Mařík, M. a kol. 2011. *Metody oceňování podniku: proces ocenění - základní metody a postupy.* 3., upr. a rozš. vyd. Praha: Ekopress.

Markowitz, H. M. 1999. "The early history of portfolio theory: 1600-1960." *Financial Analysts Journal* 55(4).

MF SR. 2016. "Výnosová krivka slovenských štátnych dlhopisov". Accessed February 9, 2016, http://www.finance.gov.sk/Default.aspx?CatID=10501.

Mossin, J. 1966. "Equilibrium in a Capital Asset Market." *Econometrica* 34(4). 768-783.

NBS. 2016. "*Štatistika dlhodobých úrokových sadzieb*" Accessed February 15, 2016. http://www.nbs.sk/sk/statisticke-udaje/menova-a-bankova-statistika/statistika-dlhodobych-urokovych-sadzieb

Pavelková, D. a A. Knápková. 2005. *Výkonnost podniku z pohledu finančního manažera*. Praha: Linde.

Petřík, T. 2009. *Ekonomické a finanční řízení firmy*. 2. vyd. Praha: Grada Publishing.

Saita, F. 2007. *Value at Risk and Bank Capital Management*. London: Academic Press.

Sharpe, W. F. 1964. "Capital Asset Prices: A theory of Market Equilibrium under Conditions of Risk." *Journal of Finance* 19(3), 425-442.

Sivák, R., Ľ. Gertler a U. Kováč. 2010. *Riziká a modely vo financiách a v bankovníctve.* 2. vyd. Bratislava: SPRINT.

Treynor, J. L. 1961. *Market Value, Time and Risk.* Unpublished manuscript.

Treynor, J. L. 1962. *Toward a Theory of Market Value of Risky Assets.*